O Level A Maths Tutorial 6: Exponential and Logarithmic Functions

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Syllabus :

- Exponential and logarithmic functions a^x , e^x , $\log_a x$, ln x and their graphs, including
  - laws of logarithms
  - equivalence of $y = a^x$ and $x = \log_a y$
  - change of base of logarithms

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1. (i) Let $y = 10^x$. Complete this table using a calculator.

x	-2	-1	0	1	2
y					

Sketch the graph.

(ii) Now switch the variable names. So the equation becomes $x = 10^y$. To make y the subject, the function is called logarithm. The short form is log, and is written like this:

$$y = \log_{10} x$$

Using a calculator, complete this table.

x	0.4	0.6	0.8	1	2	3	4
y							

Sketch the graph.

2. (a) State the value of e to 4 significant figures.

(b) Find the value of ln 10, where $\ln 10 = \log_e 10$. ($\log_e$ is called natural logarithm.)

3. Law 1 of logarithm - Product rule:

$$\log_a xy = \log_a x + \log_a y$$

Given that $\log_{10} a = 1.2$ and $\log_{10} b = 0.7$, find $\log_{10} ab$.

4. Law 2 of logarithm - Quotient rule:

$$\log_a x/y = \log_a x - \log_a y$$

Given that $\log_{10} a = 1.2$ and $\log_{10} b = 0.7$, find $\log_{10} ab$.

5. Law 3 of logarithm - Power rule:

$$\log_a x^n = n \log_a x$$

Given that $\log_{10} a = 1.2$, find $\log_{10} a^5$.

6. Law 4 of logarithm - Change of base:

$$\log_a x = \log_b x / \log_b a$$

Given that $x = 20$ and $b = 10$, find $\log_5 20$.

• Simplifying expressions and solving simple equations involving exponential and logarithmic functions

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7. (i) Solve $2^{x-1} = 8$

(ii) Solve $\log_3 x = 2$

• Using exponential and logarithmic functions as models

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8. There are 1000 bacteria in a dish. The number of bacteria doubles every 4 hours. The number of bacteria after time t is

$$N = 1000\, e^{-kt}.$$

Find k.